

**Mathematics Specialist Unit 1&2
Test 5 2018**

Calculator Free
Matrices

STUDENT'S NAME _____

DATE: Monday 20 August

TIME: 21 minutes

MARKS: 21

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

1. (4 marks)

Consider the following matrices:

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & -3 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 & 2 \\ 7 & -3 & -1 \end{bmatrix} \quad C = [2 \quad 3] \quad D = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

Determine where possible:

(a) $A - 3B$ [2]

(b) DC [2]

2. (7 marks)

Consider the following three matrices:

$$A = \begin{bmatrix} 1 & a-1 \\ 1-x & -5 \end{bmatrix}$$

$$B = \begin{bmatrix} a-1 & c \\ b+2 & d+5 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & -3 \\ -4 & 2c \end{bmatrix}$$

(a) Determine an expression for the value of x that will make matrix A singular. [3]

(b) Determine the values of a, b, c and d if $B = 2C + I$, where I is the 2×2 identity matrix. [4]

3. (5 marks)

Matrices A and B are defined as follows:

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 6 \\ 7 & x^2 \end{bmatrix}$$

(a) Determine the matrix AB [2]

(b) If $AB = \begin{bmatrix} 31 & 24 \\ 55 & 44 \end{bmatrix}$, and $x < 0$, calculate the value of x . [3]

4. (5 marks)

Determine the Cartesian equation in exact form, of a parabola, $y = x^2$, after it has been rotated 45° anticlockwise about the origin.

Note – A Cartesian equation is expressed in terms of x and y only.



**Mathematics Specialist Unit 1&2
Test 5 2018**

**Calculator Assumed
Matrices**

STUDENT'S NAME _____

DATE: Monday 20 August

TIME: 29 minutes

MARKS: 29

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5. (4 marks)

(a) If $A = \begin{bmatrix} 0 & 4 & 2 \\ 2 & 2 & 2 \\ -2 & -4 & -4 \end{bmatrix}$ determine A^2 [1]

(b) Use the result from part (a) to solve the following simultaneous equations. Show your matrix equations. [3]

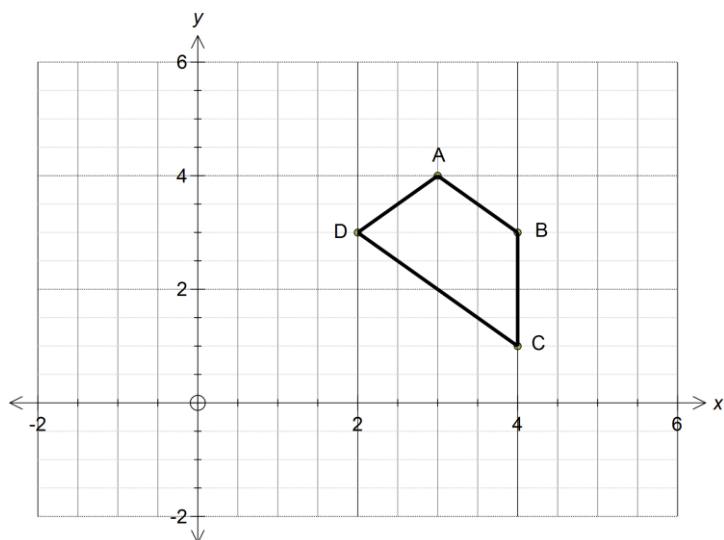
$$\begin{aligned} 4y + 2z &= -2 \\ 2x + 2y + 2z &= 0 \\ 2x + 4y + 4z &= -6 \end{aligned}$$

6. (5 marks)

Determine the matrix A , given that $A \begin{bmatrix} 6 & 5 \\ -1 & 1 \end{bmatrix} - 3A = \begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}$. Show all matrix equations.

7. (10 marks)

Jacob is opening his own tutoring business and decides to design a logo for his business. So far all he has is the logo drawn below.



Jacob decides to manipulate this logo by first transforming it using the matrix $M = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and then by using the matrix $N = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$.

(a) Describe the transformation performed by transformation matrix M . [1]

(b) Determine the image of the original points under the transformation given by M . [2]

- (c) Determine the single transformation that would give the same image as performing transformation M followed by transformation N . [2]
- (d) If the original points are transformed by the matrix found in part (c), determine:
- (i) the area of the original logo object. [1]
- (ii) the area of the new logo image. [2]
- (e) Assuming the transformation described in part (c) has taken place, determine a single matrix that would transform the new image back to the original logo. [2]

8. (6 marks)

An object undergoes the following sequence of transformations:

- reflection in the line $y = \sqrt{3}x$, then
- shear parallel to the x-axis with a scale factor of -2 , then
- rotation clockwise of 90°

(a) Determine a single transformation matrix to perform this sequence of transformations. [5]

(b) Determine which point, if any, transforms (maps) to itself. [1]

9. (4 marks)

The transformation matrix, $\begin{bmatrix} 1 & a \\ 3 & 2 \end{bmatrix}$, transforms (maps) all points to a single line.

(a) Determine the value(s) of a . [2]

(b) Determine the equation of the line [2]